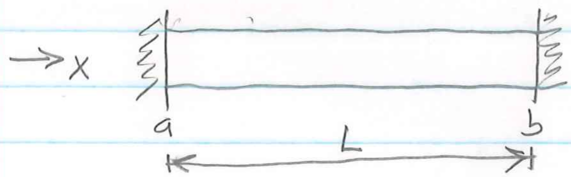
9. Temperature change

Assume the material is linearly elastic, homogeneous, isotropic. If such a body is supported in a statically determinant way, then a uniform temperature change ΔT will cause the following strains to occur: $\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha \Delta T$, $\gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0$, without producing any stresses. α must also be constant throughout the body, where α is the coefficient of thermal expansion. For the statically indeterminant case, the ΔT will cause stresses.

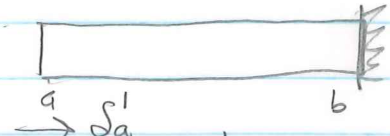
Example: The bar is fixed at both ends and ΔT is applied. Find the reaction at a. Assume only σ_x exists and is uniform across the cross-section.



E, A, α Apply ΔT

Use superposition.

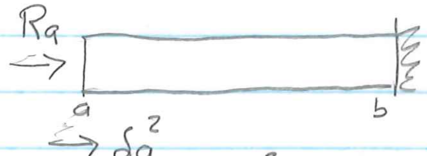
Step 1.



ΔT applied

$$\delta_a^1 = -\alpha \Delta T \cdot L$$

Step 2.



R_a applied

$$\delta_a^2 = R_a / \frac{EA}{L}$$

Step 3.

$$\delta_a^1 + \delta_a^2 = 0$$

Substitute and solve for R_a: $R_a = \alpha \Delta T \cdot EA$

(independent of L)

Example: If the bar were made of steel, what ΔT would cause the bar to yield?

$\sigma_x = R_a / A = \alpha \Delta T \cdot E = \sigma_Y$ ~ yield

For steel, $\alpha = 0.0000065$ strain/°F

$E = 30 \times 10^3$ ksi

$\sigma_Y = 36$ ksi

get $\Delta T = 185$ °F

Example: How much would a 1000 mile straight pipe line shorten for a temperature drop of 60 °F?

$\Delta L = \alpha \Delta T \cdot 1000$ miles   use α for steel

$= 2059$ feet

Or would friction between the pipe and the ground reduce this?

Example: Calculate the restraining force to anchor a steel pipe. The pipe is 10 feet in diameter with a wall thickness of 2 in. Use $\Delta T = 60$ °F.

$A = 2\pi \cdot 60 \text{ in} \cdot 2 \text{ in} = 754.0 \text{ in}^2$

restraining force $= F = \alpha \Delta T \cdot E \cdot A$ (see example previous page)

$= 8{,}822$ kips   (large!)

(does not depend on L)